Numeric Response Questions

Binomial Theorem

Q.1 When 2^{30t} is divided by 5, then find least positive remainder. $x_1 \& x_2$ then find value of $x_1 + x_2$

Q.3 In the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^{n}$, the ratio of the 7^{th} term from the beginning to the 7^{th} term from the end is $1:6$; then find n.

Q.4 If $a^2 + b = 2$ then maximum value of term independent of x in expression of (axis + bx-25) $(a > 0, b > 0)$ is $9^k + k + 1$, then find value of k.

Q.5 If the term independent of x in $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405 , then find value of k.

Q.6 Number of irrational terms in expansion of $(\sqrt{3}+\sqrt{7})^{17}$ is equal to $3k + k$ then find value of k.

Q.7 If the third term in the expansion of $\left[x + x^{\log_{10} x}\right\}^5$ is equal to 10,00,000, then $x =$

Q.8 If ${}^{\infty}C_4 + \sum_{r=1}^{6} m_{6-r}C_3 = {}^{n} + k_4$ then find k.

Q.9 If $(1 + x - 2x^2)^0 = 1 + a1x + ax^2 + \cdots . + a + 2x^{22}$, then find the value of expression a2 $+a_4 + a^2 + \cdots + a_{12}$

Q.10 If 2, ${}^{10}C_0 + \frac{2^2}{2}$, ${}^{10}C_1 + \frac{2^3}{3}$, ${}^{10}C_2 + \cdots \ldots + \frac{2}{11}^{11}$, ${}^{10}C_{10} = \frac{3^{k-1}}{11}$ then find k.

Q.11 If the coefficient of x^{-7} in $\left(2x - \frac{1}{3x^2}\right)^{11}$ is $11C_6 \frac{2^5}{3^4}$ then find k.

Q.12 The sum of the series ${}^{10}C_{1.}4 + 10C_2 4^2 + \cdots \ldots + 10C_{10} 4^{\text{to}}$ is $5^k - 1$ then find k.

Q.13 If the 17^{th} and 18^{th} terms of the expansion $(2 + a)^{ri}$ are equal, then find the value of 'a'.

Q.14 Find the coefficients of x^5 in the expansion of $(1 + x^2)^4 (1 + x)^5$.

Q.15 Find the number of rational terms in the expansion of $(7^{1\pi} + 11^{109})^{561}$.

ANSWER KEY

1. 2.00	**2.** 3.00	**3.** 9.00	**4.** 2.00	**5.** 3.00	**6.** 3.00	**7.** 10.00
8. 6.00	**9.** 31.00	**10.** 11.00	**11.** 6.00	**12.** 10.00	**13.** 1.00	**14.** 71.00
15. 730.00						

Hints & Solutions

1. $2^{301} = 2.2^{300} = 2.4^{150} = 2(5-1)^{150}$

Here all terms, except last term are divisible by 5

$\therefore$ Remainder = 2(last term) = $2(-1)^{150} = 2$

2. $$\left[2^{\log_2\sqrt{(9^{x-1}+7)}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$$

$$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

Here

$T_6 = 84$

$$\Rightarrow {}^7C_5.\left(\sqrt{9^{x-1}+7}\right)^2.\left(\frac{1}{(3^{x-1}+1)^{1/5}}\right)^5 = 84$$

$$\Rightarrow \frac{21.(9^{x-1}+7)}{(3^{x-1}+1)} = 84$$

$\Rightarrow 9^{x-1} + 7 = 4(3^{x-1} + 1)$

Put $3^{x-1} = t$

$\Rightarrow t^2 + 7 = 4t + 4$

$\Rightarrow t^2 - 4t + 3 = 0 \quad \Rightarrow t = 1$ or $t = 3$

-3^{x-1} | $3^{x-1} = 3$

$\therefore x = 1$ | $x = 2$

3. $$\left(2^{\frac{1}{3}} + \frac{1}{3^{1/3}}\right)^n$$

7^{th} term from the beginning

$$= {}^nC_6 \frac{1}{\left(3^{\frac{1}{3}}\right)^6}\left(2^{\frac{1}{3}}\right)^{n-6}$$

7^{th} term from the end = (n – 7 + 2)th term from the beginning i.e. (n – 5)th term.

Hence, $$\frac{{}^nC_6 \frac{1}{\left(3^{\frac{1}{3}}\right)^6}\left(2^{\frac{1}{3}}\right)^{n-6}}{{}^nC_{n-6} \frac{1}{\left(3^{\frac{1}{3}}\right)^{n-6}} \times \left(2^{\frac{1}{3}}\right)^6} = \frac{1}{6}$$

$$\Rightarrow \left(3^{\frac{1}{3}}\right)^{n-12} \times \left(2^{\frac{1}{3}}\right)^{n-12} = 6^{-1}$$

$$\Rightarrow \left(6^{\frac{1}{3}}\right)^{n-12} = 6^{-1}$$

$$\Rightarrow (6)^{\frac{n-12}{3}} = 6^{-1}$$

$$\Rightarrow \frac{n-12}{3} = -1$$

$\Rightarrow n - 12 = -3$

$\Rightarrow n = 9$ Ans.

4. Let T_{r+1} term is independent of x

$\therefore T_{r+1} = {}^9C_r\,(ax^{1/6})^{9-r}\,(bx^{-1/3})^r$

$$= {}^9C_r\, a^{9-r}\,.\, b^r\, x^{\frac{9-r}{6}-\frac{r}{3}}$$

$$\therefore \frac{9-r}{6} - \frac{r}{3} = 0$$

$\Rightarrow r = 3$

$\therefore T_4 = {}^9C_3\, a^6b^3$

$$= 84\left(\sqrt{a^2b}\right)^6$$

$$\le 84\left(\frac{a^2+b}{2}\right)^6$$

$(T_4)_{max.} = 84$

5. $t_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r\, x^{5-\frac{5r}{2}} (-k)^r$

For the independent of x, r must be 2 so that

${}^{10}C_2\, k^2 = 405$

$\therefore k = 3$

6. $\because (\sqrt{3}+\sqrt{7})^{17} = \sum_{r=0}^{17} {}^{17}C_r (\sqrt{3})^r (\sqrt{7})^{17-r}$

$= \sum_{r=0}^{17} {}^{17}C_r\, 3^{\frac{r}{2}}\, 7^{\frac{17-r}{2}}$

$\because$ Both $\frac{r}{2}$ & $\frac{17-r}{2}$ can't be integer at same so all term are irrational

$\therefore$ Total irrational terms are 18

$\therefore k = 3$

7. $T_3 = {}^5C_2.(x)^3.(x^{\log_{10}x})^2 = 10^6$

$x^3.\ (x^{\log_{10}x})^2 = 10^5 = 10^3.\ (10)^2$

$\Rightarrow x = 10$

8. ${}^{50}C_4 + ({}^{55}C_3 + {}^{54}C_3 + \ldots + {}^{50}C_3)$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + \ldots {}^{55}C_3$

$= {}^{52}C_4 + {}^{52}C_3 + \ldots + {}^{55}C_3$

= ....

= ....

$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

9. Put x = 1, x = – 1

$(1 + 1 - 2)^6 = 0 = 1 + a_1 + a_2 + \ldots + a_{12}$

$+\ (1 - 1 - 2)^6 = 64 = 1 - a_1 + a_2 + \ldots + a_{12}$

$64 = 2[1 + a_2 + a_4 + \ldots + a_{12}]$

$\Rightarrow a_2 + a_4 + \ldots + a_{12} = 32 - 1 = 31$

10. $\int_0^2 (1+x)^{10} = \int_0^2 {}^{10}C_0 + {}^{10}C_1.x + \ldots + {}^{10}C_{10}\, x^{10}$

$\left|\frac{(1+x)^{11}}{11}\right|_0^2 = {}^{10}C_0.(2) + {}^{10}C_1.\frac{(2)^2}{2} + {}^{10}C_2 \frac{(2)^3}{3} + \ldots + {}^{10}C_{10} \frac{(2)^{11}}{2}$

$= \frac{(1+2)^{11}-1}{11} = \frac{3^{11}-1}{11}$

11. $T_{r+1} = {}^{11}C_r . (2x)^{11-r} . \left(-\frac{1}{3x^2}\right)^r$

$(x)^{11-r-2r} = (x)^{-7}$

$11 - 3r = -7$

$3r = 18$

$r = 6$

Coeffi. $= {}^{11}C_6 . (2)^5 . \left(\frac{-1}{3}\right)^6 = {}^{11}C_6 . \frac{(2)^5}{(3)^6}$

12. $(1 + 4)^{10} - {}^{10}C_0 = 5^{10} - 1$

13. $T_{(p-1)+1} = T_p = {}^6C_{p-1}\, (2x)^{6-(p-1)}\, 3^{(p-1)}$

$T_p = {}^6C_{p-1} . 2^{7-p}\, 3^{p-1}\, x^{7-p}$

Coeff. of T_p is ${}^6C_{p-1} . 2^{7-p}.3^{p-1}$ equal to 4860

$\therefore p = 5$

14. $(1 + t^{12} + t^{24} + t^{36}) . ({}^{12}C_0 + {}^{12}C_1(t^2)^1 + \ldots$ $s + {}^{12}C_6\, (t^2)^6 + \ldots + {}^{12}C_{12}\, t^{24})$

$= {}^{12}C_6 + 1 + 1$

15. $T_{r+1} = {}^{6561}C_r\, (7^{1/3})^{6561-r}.(11^{1/9})^r$

$= {}^{6561}C_r\, 7^{2187} . 7^{-r/3} . 11^{r/9},\ 0 \le r \le 6561$

Now T_{r+1} will be rational when r/3 and r/9 both are integers. This is possible only when r is a multiple of 9. But multiple of 9 from 0 to 6561 are

0, 9, 18, 27, ………, 6561

These are in AP, so their number n is given by

$6561 = 0 + (n - 1)\, 9 \Rightarrow n = 730$